E 7121 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2017

Fifth Semester

Core Course—FUZZY MATHEMATICS

(For Model I B.Sc. Mathematics)

[2013 Admission onwards]

Time : Three Hours Maximum Marks : 80

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define Boolean lattice.
2. Define standard complement of a fuzzy set A.
3. Define α-cut of a fuzzy set A.
4. Give an example of a strong cutworthy property.
5. State De Morgan's laws in crisp sets.
6. Define axiomatic skeleton for fuzzy t-conorms.
7. Give an example of a continuous fuzzy complement.
8. Find [4, 10] / [1, 2].
9. Define a fuzzy number.
10. Define logic operations.

(10 × 1 = 10)

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Compute the scalar cardinality of $A(x) = \frac{x}{x+1}$ for $x = \{0, 1, 2, \ldots 10\} = X$.

12. Find the strong α-cut of $A = \left(\frac{1}{1+10x}\right)^{1/2}$ where $\alpha = .4$.

13. Define the degree of subsethood of A in B.

14. Let A and B are fuzzy sets. Prove that ${}^{\alpha+}A \subseteq {}^{\alpha}A$.

Turn over

15. Prove that $C_\lambda(a) = \frac{1-a}{1+\lambda a}$ is a fuzzy complement.
16. State the characterization theorem of fuzzy complements.
17. Define idempotent t-norm. Give an example of an idempotent t-norm.
18. Define dual triple. Give an example.
19. Show that $E(x) = \begin{cases} 1 & \text{for } x = 5 \\ 0 & \text{otherwise} \end{cases}$ is a fuzzy number.
20. Is every fuzzy set, a fuzzy number ? Justify.
21. Define logic formulas.
22. What are inference rules ?

(8 × 2 = 16)

Part C (Descriptive/Short Essay Type Questions)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Let A and B are fuzzy sets. Then for all $\alpha \in [0, 1]$ prove that $A \subseteq B$ iff ${}^{\alpha}A \subseteq {}^{\alpha}B$.
24. Let A and B be fuzzy sets defined on a universal set X. Prove that $|A| + |B| = |A \cup B| + |A \cap B|$.
25. Let A and B are fuzzy sets on X. Then prove that (i) ${}^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$; and (ii) ${}^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$.
26. Prove that every fuzzy complement has at most one equilibrium.
27. Let $\langle i, u, c \rangle$ be a dual triple that satisfies the law of excluded middle and law of contradiction. Then prove that $\langle i, u, c \rangle$ does not satisfy the distributive law.
28. Let i_w denote the class of Yager t-norms. Then prove that $i_{\min}(a, b) \leq i_w(a, b) \leq \min(a, b)$.
29. Do the arithmetic operations on closed intervals satisfy subdistributivity and distributivity ? Justify.
30. Explain Linguistic hedges.
31. Explain unconditional and qualified propositions.

(6 × 4 = 24)

Part D (Long Essays)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. State and prove the second decomposition theorem on fuzzy sets.
33. Given an involutive fuzzy complement c and an increasing generator g of c, prove that the t-norm and t-conorm generated by g are dual with respect to c. Also show that the fuzzy operations i, u, c satisfy the law of excluded middle and the law of contradiction.

34. Let

$$A(x) = \begin{cases} \frac{x+2}{2} & -2 < x \leq 0 \\ \frac{2-x}{2} & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

$$B(x) = \begin{cases} \frac{x-2}{2} & 2 < x \leq 4 \\ \frac{6-x}{2} & 0 < x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

Find A + B, A – B, B – A, A . B and A/B.

35. Explain fuzzy quantifiers in detail.

(2 × 15 = 30)